transonic phenomena are strongest at low reduced frequencies and decrease rapidly if the frequency is increased.

Acknowledgements

This Note summarizes work carried out under contract for the Netherlands Agency for Aerospace Programs. The authors are grateful to NASA Ames, and in particular to W.F. Ballhaus, for making LTRAN2 available to NLR.

References

[1] Ballhaus, W. F. and Goorjian, P. M., "Implicit Finite Difference Computations of Unsteady Transonic Flows about Airfoils, Including the Treatment of Irregular Shock Wave Motions," AIAA Paper 77-205, 1977.

[2] Houwink, R. and van der Vooren, J., "Some Results of an Improved Version of LTRAN2 for Computing Unsteady Airloads on Airfoils Oscillating in Transonic Flow," AIAA Paper 79-1553, 1979.

[3] Couston, M. and Angélini, J. J., "Solution of Non-Steady Two-Dimensional Transonic Small Disturbances Potential Flow Equations," ONERA T.P. 1978-69, 1978.

[4] Rizetta, D. P. and Chin, W. C., "Effects of Frequency in Unsteady Transonic Flow," *AIAA Journal*, Vol. 17, July 1979, pp. 779-781.

J80-192

Review of the Influence of Cooled Walls on Boundary-Layer Transition

20004

J. Leith Potter*

Sverdrup/ARO, Inc., Arnold Air Force Station, Tenn.

Nomenclature

M	= Mach number
Re	= Reynolds number based on wetted length
$\bar{R}e$	$= Re_{\delta t}/Re_{\delta ta}$
Taw	= surface temperature for an insulated wall, i.e., adiabatic recovery temperature
Tw	= surface temperature
$\bar{T}w$	$= Tw/Taw$
$(U/\nu)_\delta$	= unit Reynolds number at edge of boundary layer

Subscripts

a	= adiabatic wall conditions
s	= complete boundary-layer stabilization
t	= boundary-layer transition
δ	= edge of boundary layer conditions
∞	= freestream conditions

THIS Note presents a review of the influence of cooled walls on boundary-layer transition. The purpose is to clarify a somewhat confused picture which has evolved because the relevant data have been collected piecemeal for roughly three decades and are characterized by some gaps in coverage as well as disagreements. Although a completely coherent view is not developed here, it is believed that some improvement in understanding is gained by looking at the data in as general a framework as possible.

Figure 1 is the basis for most of the following discussion. It displays the available experimental data on the influence of cooled walls on the Reynolds number of boundary-layer transition under supersonic flow conditions.[1-8] Only results which include the condition $Tw = Taw$ are presented because of a need to normalize the data. In selecting data to be presented, it was necessary to accept sources where different methods of determining $Re_{\delta t}$ were used, and presumably different combinations of flow disturbances were present in the various wind tunnels. Only data for zero pressure gradient are represented, but both sharp cones and sharp plates or wedges are included. At a given M_δ, data for a constant $(U/\nu)_\delta$ were utilized, but different $(U/\nu)_\delta$ had to be accepted at different M_δ. There is evidence that the influence of wall cooling varies somewhat with unit Reynolds number at a given Mach number (cf. Refs. 5 and 7). Freestream temperature also generally varied with Mach number. A significant step toward eliminating the effects of such variables was taken when the ratio $\bar{R}e$ was adopted. The use of this ratio also makes possible a more simple and orderly display of the various data included in Fig. 1.

Unfortunately, the adiabatic wall case $\bar{T}w = 1$ often is not included in investigations of higher Mach number flows, which prevents inclusion of such data in Fig. 1. Those cases are discussed later. In regard to Ref. 5, the data do not extend to $\bar{T}w = 1$, but extrapolation to obtain $Re_{\delta ta}$ seemed safe enough to justify that being done. In Ref. 8, only a few points are given and a claim could be made that no $\bar{T}w$ effect is evident.

In addition to the Ref. 4 data shown in Fig. 1, there are other experimental results which apparently exhibit no $\bar{T}w$ effect. Because $Re_{\delta ta}$ is not available in those cases, they are not represented in Fig. 1, but they must be recognized at this point. References 9-11 are the principal ones that the writer is aware of. Sanator et al.[9] covered the range $0.08 \leq \bar{T}w \leq 0.4$ at $M_\delta = 8.8$ and found no significant change in $Re_{\delta t}$ on a sharp cone. The adiabatic wall condition could not be included. Deem and Murphy[10] also report no effect for $0.2 \leq \bar{T}w \leq 0.8$ at $M_\delta = 10.2$, and Everhart and Hamilton[11] found no effect for $0.4 \leq \bar{T}w \leq 0.6$ at $M_\delta = 8.9$.

In contrast to the findings of the last three sets of investigations, there are references showing marked $\bar{T}w$ influence at hypersonic Mach numbers. Stetson and Rushton[12] reported a pronounced decrease of $Re_{\delta t}$ on a cone as $\bar{T}w$ decreased in the region $0.2 \leq \bar{T}w \leq 0.6$ at $M_\delta = 4.8$. This rather strongly suggests that a transition reversal would have been found somewhat above $\bar{T}w = 0.6$ if it had been feasible to explore higher $\bar{T}w$. Mateer[13] has reported a similar decrease of $Re_{\delta t}$ with decreasing $\bar{T}w$ for $0.2 \leq \bar{T}w \leq 0.4$ and an indication of a reversal at $\bar{T}w \approx 0.2$, so that $Re_{\delta t}$ began increasing again as $\bar{T}w$ was lowered below 0.2. His data

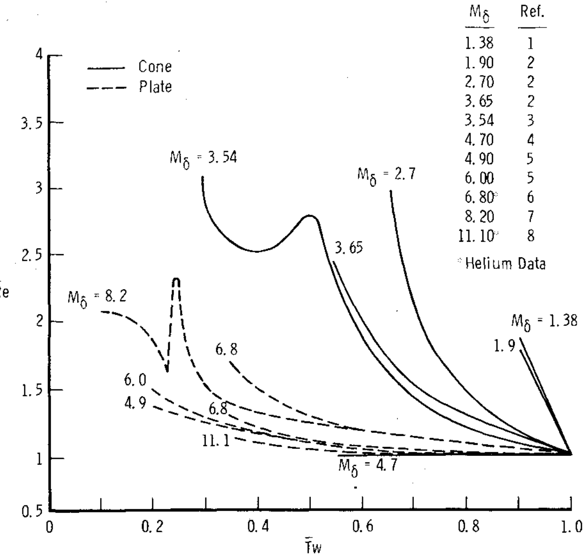

Fig. 1 Experimental effect of wall cooling on transition Reynolds numbers at various Mach numbers.

Received Oct. 12, 1979; revision received Feb. 8, 1980.

Index category: Boundary-Layer Stability and Transition.

*Deputy for Technology, AEDC Division. Fellow AIAA.

correspond to $M_\delta = 5$ and 6.6 on a cone. Sheetz[14] and Reda[15] both studied cones in an aeroballistics range and report multiple reversals, i.e., $Re_{\delta t}$ increasing or decreasing in different ranges of $\bar{T}w$ for various M_δ between 4.2 and 6.9.

Only a loose synthesis of these sometimes contradictory data can be considered. It appears that all reports referenced herein for $M_\delta \lesssim 4$ are consistent in the conclusion that there was an increase in $Re_{\delta t}$ as $\bar{T}w$ decreased from unity. This behavior is consistent with the theory of boundary-layer stability as developed to the present day, cf. Refs. 16-18 and Refs. 19 and 20. The rate of increase is great at the lower supersonic Mach numbers and declines, but remains quite strong up to at least $M_\delta \approx 4$. Thereupon, if the data presented in Fig. 1 are taken literally, a rather rapid decline in the effect of $\bar{T}w$ is found. However, there is no reason to necessarily assume monotonic behavior of $\bar{R}e$ ($\bar{T}w$) with increasing M_δ; indeed, more recent boundary-layer stability analyses[16-20] suggest a complex interplay of transition factors in the hypersonic regime. There is also the knowledge that slightly different levels of $\bar{R}e$ result at different unit Reynolds numbers, as illustrated by the two curves for $M_\delta = 6.8$ in Fig. 1 where the upper curve corresponds to a lesser value of unit Reynolds number.

The $\bar{R}e$ curves for $M_\delta = 3.54$ and 8.2 exhibit a reversal, a drop, and then once again a rising trend in $\bar{R}e$ as $\bar{T}w$ decreases further. By inference, it must be accepted that other data may have behaved in a qualitatively similar manner if lower $\bar{T}w$ could have been investigated. At least the possibility of single or even multiple reversals must be recognized, and, of course, the roles of other factors must not be overlooked. Surface roughness, in particular, becomes more critical as $\bar{T}w$ decreases, thinning the boundary layer and increasing local unit Reynolds numbers in the wall region. It is easy to visualize $\bar{R}e$ increasing as $\bar{T}w$ decreases, until the effect of roughness becomes strong enough to cause $\bar{R}e$ to begin decreasing with the further lowering of $\bar{T}w$. Reference 2 illustrates such an event, and it is analyzed in Ref. 21. However, one cannot now say that the "first" reversals of $\bar{R}e$ are always caused in that manner. It would seem to require greater roughness than was present, unless unobserved frost formed in the experiments where reversals occurred and the re-reversal (increase in $\bar{R}e$) sometimes observed at still lower $\bar{T}w$ is hard to reconcile with a roughness-dominated interaction.

It is well known that the linear theory of boundary-layer stability predicts only a favorable influence of cooling for supersonic Mach numbers. Examination of theoretical predictions in Refs. 19 or 22 tempts one to conclude that $\bar{R}e$ rises as $\bar{T}w$ drops below unity, until the latter reaches a level roughly approximating the value of $\bar{T}w$ theoretically predicted to produce complete stabilization of two-dimensional, first-mode disturbances. A possible reason for reversal of that trend is found in results of more complex calculations[16-20] which show that cooling should not be sufficient to assure complete stability where higher-mode disturbances become dominant over the first mode. Therefore, it may be that the first reversal of transition occurs when cooling no longer is adequate to assure stability, and higher modes, unstabilized by cooling, have taken over. If that is true, an explanation of the experimentally observed re-reversal at still lower $\bar{T}w$ must await further research. There is, of course, the possibility that the explanation may involve either or both undetected frost formation and inadequately understood instability modes. The difficulty of avoiding frost on highly cooled surfaces in transition studies is well known.

Perhaps it should be noted that the results of Ref. 23 cover a range of Mach numbers at various $\bar{T}w$ and exhibit numerous transition trend reversals. However, the difficulty of relating those shock tube experiments to the wind-tunnel data discourages comparison.

A promising area for realizing the higher $\bar{R}e$ (not necessarily $Re_{\delta t}$) resulting from wall cooling to levels near $\bar{T}ws$ appears to correspond to $1.5 \lesssim M_\delta \lesssim 4$. Very high $Re_{\delta t}$ was reported by Sternberg[24] at $M_\delta = 2$, and rather high values were also reported in Refs. 25-27, all achieved by wall cooling under conditions where the deleterious effects of other factors apparently were minimal. Under free-flight conditions, with zero pressure gradient at $M_\delta \approx 2$, Sternberg's data show $Re_{\delta t} \approx 40 \times 10^6$ for the beginning of transition on a sharp cone. This exceptionally large value of $Re_{\delta t}$ was achieved with $\bar{T}w \approx \bar{T}ws$. Czarnecki and Sinclair[25] attained $Re_{\infty t} \approx 28 \times 10^6$ by cooling a parabolic body of revolution to $\bar{T}ws$ in a wind tunnel at $M_\infty = 1.61$. Thus in the absence of interference from other vehicle components, with a very smooth wall, and perhaps a little luck, something approximating $Re_{\delta t} = 30\text{-}40 \times 10^6$ may be achieved by cooling to $\bar{T}ws$ for two-dimensional or axisymmetric bodies at low supersonic Mach numbers. However, the practical obstacles to be overcome in routine flight operations are obvious.

The free-flight aeroballistic range data of Refs. 14, 15, 28, and 29 do not display exceptionally high $Re_{\delta t}$ values, but correspond to very low $\bar{T}w$ relative to the Mach numbers investigated. Therefore, there is a good chance that first reversals or peaks in $Re_{\delta t}$ would have been found at higher $\bar{T}w$ if it had been feasible to extend the range of $\bar{T}w$ up to values typical of wind-tunnel experiments.

In conclusion, it is clear that the effect of $\bar{T}w$ on $\bar{R}e$ is strongly dependent on M_δ and probably weakly dependent on $(U/\nu)_\delta$. At $M_\delta \lesssim 4$, and possibly higher, the experimentally observed effect is qualitatively in agreement with the stability theory in the area of initial departure of $\bar{T}w$ from unity. The Mach number restriction could be lifted if one or two sets of data were ignored, but there is no apparent justification for that. The greatest practically attainable values of $\bar{R}e$ appear to lie in the lower supersonic Mach number regime, say $1.5 \lesssim M_\delta \lesssim 4$, but secondary flows or unfavorable interference from vehicle components obviously could prevent the achievement of high $Re_{\delta t}$ by wall cooling in more complex flows.

References

[1] Sherrer, R., "Boundary Layer Transition on a Cooled 20° Cone at Mach Numbers of 1.5 and 2.0," NASA TN 2131, 1950.

[2] Van Driest, E. R. and Boison, J. C., "Experiments on Boundary Layer Transition at Supersonic Speeds," *Journal of the Aeronautical Sciences*, Vol. 24, Dec. 1957, pp. 885-899.

[3] Wisniewski, R. J. and Jack, J. R., "Recent Studies on the Effect of Cooling on Boundary Layer Transition at Mach 4," *Journal of the Aeronautical Sciences*, Vol. 28, March 1961, pp. 250-251.

[4] Krogmann, P., "Experimental Study of Boundary Layer Transition on a Slender Cone at Mach 5," AGARD Symposium on Laminar-Turbulent Transition, Technical University of Denmark, Copenhagen, May 2-4, 1977.

[5] Cary, A. M., "Turbulent Boundary Layer Heat Transfer and Transition Measurements for Cold-Wall Conditions at Mach 6," *AIAA Journal*, Vol. 6, May 1968, pp. 958-959.

[6] Maddalon, D. V., "Effects of Varying Wall Temperature and Total Temperature on Transition Reynolds Number at Mach 6.8," *AIAA Journal*, Vol. 7, Dec. 1969, pp. 2355-2357.

[7] Richards, B. E. and Stollery, J. L., "Further Experiments on Transition Reversal at Hypersonic Speeds," *AIAA Journal*, Vol. 4, Dec. 1966, pp. 2224-2226.

[8] Watson, R. D., "Wall Cooling Effects on Hypersonic Transitional/Turbulent Boundary Layers at High Reynolds Numbers," *AIAA Journal*, Vol. 15, Oct. 1977, pp. 1455-1461.

[9] Sanator, R. J., DeCarlo, J. P., and Torrillo, D. T., "Hypersonic Boundary Layer Transition Data for a Cold-Wall Slender Cone," *AIAA Journal*, Vol. 3, April 1965, pp. 758-760.

[10] Deem, R. E. and Murphy, J. S., "Flat-Plate Boundary Layer Transition at Hypersonic Speeds," USAF Flight Dynamics Lab., FDL-TDR-64-129, Oct. 1964.

[11] Everhart, P. E. and Hamilton, H. H., "Experimental Investigation of Boundary Layer Transition on a Cooled 7.5° Total-Angle Cone at Mach 10," NASA TN D-4188, Oct. 1967.

[12] Stetson, K. F. and Rushton, G. H., "Shock Tunnel Investigation of Boundary-Layer Transition at $M = 5.5$," *AIAA Journal*, Vol. 5, May 1967, pp. 899-906.

[13] Mateer, G. C., "Effects of Wall Cooling and Angle of Attack on Boundary Layer Transition on Sharp Cones at $M_\infty = 7.4$," NASA TN D-6908, Aug. 1972.

[14] Sheetz, Jr., N. W., "Ballistic Range Boundary-Layer Transition Measurements on Cones at Hypersonic Speeds," *Symposium on Viscous Drag Reduction*, Plenum Press, New York, 1969, pp. 53-83.

[15] Reda, D. C., "Boundary-Layer Transition Experiments on Sharp, Slender Cones in Supersonic Freeflight," Naval Surface Weapons Center/WOL, TR-77-59, Sept. 30, 1977.

[16] Mack, L. M., "Linear Stability Theory and the Problem of Supersonic Boundary-Layer Transition," *AIAA Journal*, Vol. 13, March 1975, pp. 278-289.

[17] Mack, L. M., "Boundary Layer Stability Theory," Jet Propulsion Lab. Rept. 900-277, Rev. A, Nov. 1969.

[18] Mack, L. M., "Effects Due to Heat Transfer and Compressibility," *Boundary Layer Theory*, edited by Hermann Schlichting, 7th ed., Chap. XVII, McGraw-Hill, New York, pp. 514-525.

[19] Reshotko, E., "Transition Reversal and Tollmien-Schlichting Instability," *The Physics of Fluids*, Vol. 6, March 1963, pp. 335-342.

[20] Reshotko, E., "Boundary Layer Stability and Transition," in *Annual Review of Fluid Mechanics*, Vol. 8, edited by M. Van Dyke, W. G. Vincenti, and J. V. Wehausen, Annual Reviewers, Inc., 1976, pp. 311-342.

[21] Potter, J. L. and Whitfield, J. D., "The Relation Between Wall Temperature and the Effect of Roughness on Boundary Layer Transition," *Journal of the Aeronautical Sciences*, Vol. 28, Aug. 1961, pp. 663-664.

[22] Van Driest, E. R., "Calculation of the Stability of the Laminar Boundary Layer in a Compressible Fluid on a Flat Plate with Heat Transfer," *Journal of the Aeronautical Sciences*, Vol. 19, Dec. 1952, pp. 801-812.

[23] Boison, J. C., "Investigation of Test Facility Environmental Factors Affecting Boundary Layer Transition," USAF Flight Dynamics Lab., TR-73-106, Sept. 1973.

[24] Sternberg, J., "A Free-Flight Investigation of the Possibility of High Reynolds Number Supersonic Laminar Boundary Layers," *Journal of the Aeronautical Sciences*, Vol. 19, Nov. 1952, pp. 721-733.

[25] Czarnecki, K. R. and Sinclair, A. R., "Effects of Heat Transfer on Boundary-Layer Transition on a Parabolic Body of Revolution," NACA Rept. 1240, Feb. 1953.

[26] Rumsey, C. B. and Lee, D. B., "Measurements of Aerodynamic Heat Transfer and Boundary Layer Transition on a 10° Cone in Free Flight at Supersonic Mach Numbers up to 5.9," NACA TN D-745, 1961.

[27] Rumsey, C. B. and Lee, D. B., "Measurements of Aerodynamic Heat Transfer and Boundary Layer Transition on a 15° Cone in Free Flight at Supersonic Mach Numbers up to 5.2," NACA TN D-888, 1961.

[28] Potter, J. L., "Boundary-Layer Transition on Cones Near Mach One in an Aeroballistic Range," AEDC-TR-74-115, Jan. 1975.

[29] Potter, J. L., "Boundary-Layer Transition on Supersonic Cones in an Aeroballistic Range," *AIAA Journal*, Vol. 13, March 1975, pp. 270-277.

J80-193

Indicial Approach to Harmonic Perturbations in Transonic Flow

0009
0018

D. E. Davies* and Deborah J. Salmond†
Royal Aircraft Establishment, Farnborough, England

I. Introduction

BALLHAUS and Goorjian[1] suggest that harmonic aerodynamic force coefficients in transonic flow be calculated from the time-dependent loading generated following a step change in angle of attack. Their upwash involves a step change with time, and this step change has to be modeled in a finite difference approximation to the equations of motion. The step change in upwash is regarded as causing a perturbation to the steady transonic flow and, although the governing equations of the flow are nonlinear, their procedure for obtaining the harmonic force coefficients is strictly valid only if the perturbation constituent of the flow may be regarded as being governed by linearized equations. Rizzetta and Chin[2] also determine the loading generated following a step change in angle of attack and, in contrast to Ballhaus and Goorjian,[1] they do get an abrupt change with time in the loading. Ballhaus and Goorjian[1] make a low-frequency approximation in the governing differential equation of transonic flow, whereas Rizzetta and Chin[2] do not.

The step change in upwash is rather severe for a finite difference approximation to cope with, and more accurate results may be anticipated if an upwash which changes more gently with time is used. The purpose of this Note is to suggest a series of airfoil motions which give upwashes which are as smooth functions of time as one desires for application of a finite difference approximation. Also, it is shown how the harmonic aerodynamic force coefficients may be calculated from the aerodynamic loadings corresponding to these upwashes when the perturbation motion is governed by linearized aerodynamics.

II. General Considerations

In a modal approach, the normal displacement $Z_j(x,t)$ in mode j of a point x on the airfoil at time t in an arbitrary oscillation may be written as

$$Z_j(x,t) = cf_j(x)q_j(t) \tag{1}$$

where c is the airfoil chord, $f_j(x)$ is the jth modal function, and $q_j(t)$ is the jth generalized coordinate. If the oscillating airfoil is immersed in an airstream of density ρ and of speed V in the direction of the x axis, the airfoil experiences a loading distribution $L_j(x,t)$ (pressure force per unit area) which may be written as

$$L_j(x,t) = \rho V^2 \ell_j(x,t) \tag{2}$$

For application of Lagrange's equations of motion to an airfoil motion, the generalized air force $K_{jk}(t)$ defined by

$$K_{jk}(t) = \frac{1}{c}\int_0^c \ell_j(x,t) f_k(x)\,\mathrm{d}x \tag{3}$$

is required.

If

$$q_j(t) = \delta(t) \tag{4}$$

where $\delta(t)$ is Dirac's delta function, then we write

$$K_{jk}(t) = Q_{jk}(t) \tag{5}$$

where $Q_{jk}(t)$ is the particular form of $K_{jk}(t)$ for this case.

If

$$q_j(t) = e^{i\omega t} \tag{6}$$

then we write

$$K_{jk}(t) = \bar{Q}_{jk}(\omega)\, e^{i\omega t} \tag{7}$$

The harmonic generalized air force coefficients $\bar{Q}_{jk}(\omega)$ are required for aeroelastic analysis. We can show that the function $\bar{Q}_{jk}(\omega)$ is the Fourier transform of $Q_{jk}(t)$ by

Received Dec. 3, 1979.
Index categories: Transonic Flow; Nonsteady Aerodynamics.
*Principal Scientific Officer.
†Scientific Officer.